April 7, 1993 WOOMAL VARSITY MEET

ROUND I: Elementary number theory

ALL ANSWERS MUST BE EXPRESSED AS POSITIVE INTEGERS

1. 11000 is an integer written in base b. The integer immediately preceeding it is 10222 in base b. What number is b?

2. What number is one more than triple the difference between the least common multiple and greatest common factor of 24, 56, and 96?

3. Find the sum of the prime factors of the number represented by $2^{12}-2^{11}+2^{10}-2^{9}+\cdots+2^{2}-2$.

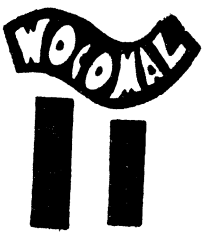
ANSWERS

(1 pt) 1. $b =$ _____

(2 pts) 2. _____

(3 pts) 3. _____

Algonquin. Burncoat. Doherty



April 7, 1993 WOJOMAL VARSITY MELT

ROUND II: Algebra 1 - open

ALL ANSWERS MUST BE EXPRESSED AS POSITIVE INTEGERS

1. If $(3^4)(2^6) = 4(6^k)$, find k.

2 Two numbers are in the ratio 5:8. If the first number is decreased by 1 and the second number is increased by 8, the new ratio is 1:2. Find the larger of the new numbers.

3 It is claimed that a new set of spark plugs, costing $13.50, will increase the mileage of a certain car from 20 to 23 miles per gallon. With gasoline at $1.20 per gallon, how many miles must be driven for the savings to pay for the new spark plugs?

ANSWERS

(1 pt) 1. _____

(2 pts) 2. _____

(3 pts) 3. _____ miles

Notre Dame, Algonquin, West Boylston

April 7, 1993 WOODMAL VARSITY MEET

ROUND III: Theory of polynomial equations and functions

ALL ANSWERS MUST BE EXPRESSED IN SIMPLEST EXACT FORM

1 Simplify $2i + \dfrac{1}{2i + \frac{1}{i}}$, where $i = \sqrt{-1}$.

2. For what value of p will $3x^4 - px + 9$ be divisible by $x-3$?

3. If $f(x+1) = x^3 - x^2 + x + 1$, express $f(x-1)$ in the form $Ax^3 + Bx^2 + Cx + D$.

ANSWERS

(1 pt) 1. _____

(2 pts) 2. _____

(3 pts) 3. _____

Algonquin, Doherty, Hudson

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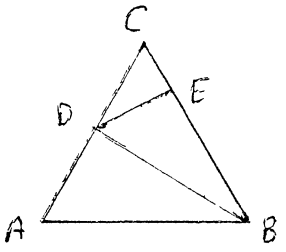
ROUND IV: Similarity and Pythagorean relationships

ALL ANSWERS MUST BE EXPRESSED IN SIMPLEST EXACT FORM OR AS DECIMALS ROUNDED TO FOUR DECIMAL PLACES

1. The lengths of the sides of a triangle are 3, 4, and 6. Find the least possible perimeter of a similar triangle having a side of length 12.

2. 

$\triangle ABC$ is equilateral with side 2.

$\triangle ABD \sim \triangle DBE$.

Find EB.

3. The medians of a right triangle which are drawn from the vertices of the acute angles have lengths 5 and $\sqrt{40}$. Find the length of the hypotenuse

ANSWERS

(1 pt) 1. _____

(2 pts) 2. _____

(3 pts) 3. _____

Bromfield, Quabrag, South

April 7, 1993 WOODMAL VARSITY MEET

ROUND V: Trigonometry - Open

ALL ANSWERS MUST BE EXPRESSED IN SIMPLEST EXACT FORM OR AS DECIMALS ROUNDED TO FOUR DECIMAL PLACES

1 Simplify to a number : $\sec 60^\circ \tan 135^\circ - \cot 60^\circ \sin 60^\circ$

2. Find all x in degrees for which $0^\circ \le x \le 360^\circ$ and $\sin x \cos x = \frac{1}{2}$.

3. For $0 \le x \le \frac{\pi}{3}$, in how many points do the graphs of $y = 2\sin(100x)$ and $y = 1$ intersect?

ANSWERS

(1 pt) 1. _____

(2 pts) 2. _____

(3 pts) 3. _____

Doherty, Millbury, St. John's

April 7, 1993 Wocomal Varsity Meet

TEAM ROUND: Topics of previous rounds and open 2 points each

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM OR AS DECIMALS ROUNDED TO FOUR DECIMAL PLACES AMD ON THE SEPARATE TEAM ANSWER SHEET

1. Find the quotient of 363_7 divided by 112_5 and write the answer in base 2

2. If a car travels for 2 miles at 20 miles/hr and for 8 miles at 40 mi/hr, what is its average speed for that time to go 10 miles in miles/hr?

3. The equation $x^2 + bx + c = 0$ has two positive integral roots r_1 and r_2. The product of the greatest common factor of r_1 and r_2 and the least common multiple of r_1 and r_2 is 216. $r_1 = \frac{3}{2} r_2$. Find the numerical value of b.

4. In trapezoid $ABCD$, $\overline{AB} \parallel \overline{CD}$, $\overline{AC} \perp \overline{BC}$, and $\overline{BD} \perp \overline{AD}$. If $AB = 25$, $AD = 15$, $BC = 15$, Find DC.

5. Evaluate $\cos^2 1^\circ + \cos^2 2^\circ + \cos^2 3^\circ + \cdots + \cos^2 89^\circ + \cos^2 90^\circ$

6. Make the line on the answer sheet a number line and graph this inequality.

$$\frac{5x}{x-1} < 4.$$ You must supply any endpoint coordinates.

7 On a simple pan balance 3 equal weight apples and 1 banana exactly balance 10 equal weight plums. Also, one apple and 6 of the plums balance the banana. How many of these plums will balance the banana?

8. 3, B, C, 123, E are five consecutive terms in a sequence of numbers in which each number beyond the second is three times the sum of the two preceeding numbers. Find E.

9 Evaluate $(987654321)(987654321) - (987654323)(987654319)$

Algonquin, Bartlett, Bromfield, Hudson, Quaboag, South, Tahanto, West Boylston

April 7, 1993

ROUND I

1. 3
2. 1993
3. 30

ROUND II

1. 4
2. 48
3. 1725

ROUND III

1. i
2. 84
3. $x^3-7x^2+17x-13$

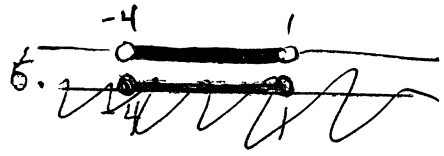
ROUND IV

1. ~~20~~ 26
2. $1\frac{1}{2}$ or...
3. $2\sqrt{13}$ or 7.211

ROUND V

1. $-2\frac{1}{2}$ or...
2. $45°, 225°$
3. 34

TEAM ROUND

1. 110_2 or 110
2. $33\frac{1}{3}$ or...
3. -30
4. 7
5. $44\frac{1}{2}$ or...
6. 

7. 7
8. 468
9. 4